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Unsteady Lifting Case by Means of the Interior-Singularity Panel Method

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This study investigates the problem of unsteady potential flow around a lifting three-dimensional wing by using the interior-singularity method. The method distributes uniform strength sources and dipoles over the panels defined on the mean plane of the lifting body. The wake surface was composed of a geometrically continuous set of quadrilateral uniform strength dipole sheets. Both the location of the wake surface and the strength of singularities are unknown. The Kutta condition was generalized to a three-dimensional unsteady case. The advantage of relating the Kutta condition to the dipole derivative is that the result converged easily. The final result shows the generated circulation and the rolling up of the trailing vortex wake. This study has shown the adaptability of the interior-singularity panel method to the three-dimensional unsteady flow problem.

Nomenclature

A	= aspect ratio $= b/c$
B	= interior singularity plane, the mean plane
b	= wing-tip to wing-tip distance
C_l	= multiplier in Eq. (15)
c	= chord length, the characteristic length
grad	= gradient operator
$\boldsymbol{i}, \boldsymbol{j}$	= unit vector on x, y axis, as shown in Fig. 2
$\boldsymbol{i}', \boldsymbol{j}', \boldsymbol{n}$	= unit vector of local coordinates lying on the wake surface. $\boldsymbol{n}$ is the surface normal; $\boldsymbol{i}'$, $\boldsymbol{j}'$ lie in x', y' direction of Fig. 1
P	= general point in the flowfield
Q	= general point lying on the singularity surface
Q_l, Q_u	= two points lying on the lower and upper sides of vortex sheet, as shown in Fig. 1
$\boldsymbol{q}$	$= \boldsymbol{U}_\infty + \boldsymbol{V}$
$\boldsymbol{q}_{\text{av}}$	$= \frac{1}{2}(\boldsymbol{q}_l + \boldsymbol{q}_u)$
$\boldsymbol{q}_l$, $\boldsymbol{q}_u$	= nondimensional velocity vectors at points Q_l and Q_u
q_l, q_u	= magnitude of $\boldsymbol{q}_l$, $\boldsymbol{q}_u$
$\boldsymbol{r}$	= position vector
$\boldsymbol{r}_0$	= position vector indicating position of the trailing edge
r	= distance from the field point to the point on singularity surface
S	= singularity surface
TE	= trailing edge
t	= nondimensional time $= U_\infty \tau / c$
$\boldsymbol{U}_\infty$	= freestream velocity, the reference velocity $\lvert \boldsymbol{U}_\infty \rvert = 1$
$\boldsymbol{V}$	= nondimensional perturbation velocity vector
$\boldsymbol{V}_\Gamma$	= velocity induced by the unit strength vortex ring
W	= wake sheet surface
x,y,z	= Cartesian coordinates shown on Fig. 2
x',y',z'	= local coordinates shown on Fig. 1
α	= angle of attack
Γ	= circulation generated
Γ_s	= circulation on the lifting body in steady flow
ΔS	= area of each singularity panel
Δt	= increment of t
$\Delta x', \Delta y'$	= finite distances used in Eq. (19)
δx	= incremental distance
μ	= dipole strength
ξ, η	= nondimensional coordinates $= x/c$ and $2y/b$, respectively
σ	= source strength
τ	= dimensional time
ω	= surface vorticity vector

Subscripts

j	= number of the strip
k	= count of the time step
l	= lower surface
u	= upper surface
KP	= Kutta point

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I. Introduction

THE steady, inviscid flow around three-dimensional bodies has been the subject of many previous studies. Either the surface-singularity method, such as by Hess,[1] or Hunt,[2] or the classic linear theory method, such as by Ashley et al.[3] or Landahl and Stark,[4] can be used for the steady flow case. Both involve assumptions on the Kutta condition and the geometric shape of the wake. The last assumption usually takes the wake to be planar for simplicity. The concept of the planar, rigid wake in the projected plane of the wing needs careful examination when the downwash velocity associated with the generation of lift is large compared with the flight velocity. A more detailed study concerning the rolling-up of the wake was conducted for the unsteady problem by Giesing,[5-7] and by Basu and Hancock[8] for the two-dimensional case. Giesing[5] investigated the effect of body thickness and the unsteady deforming wake on the lift coefficient to compare with linear theory. The method of Giesing was extended to two bodies[6] to examine the interference effect on the deformation of the wake and on the lift and drag. In Giesing,[7] the Kutta condition is related to the two-dimensional vorticity shed at the trailing edge. A conformal-mapping technique was used to compute velocity distribution near the trailing edge. The difference between this and Giesing's[5,6] equal pressure Kutta condition was also discussed. Basu and Hancock[8] derived the Kutta condition from the circulation about a symmetric airfoil, then utilized the surface-singularity method to calculate the pressure distribution on the airfoil and trailing vortex locations.

For the three-dimensional unsteady problem, a lifting method of calculating the unsteady flow about a thin wing was studied by Belotserkovskii.[9] Because Belotserkovskii's method is not capable of determining the geometry of the wake, it is limited to small angles of attack. Atta et al.[10] and Kandil et al.[11] examined the sharp-edge separation of the wake, which was represented by families of discrete vortex lines. The method extended the representation to a large angle of attack and a small aspect ratio for the thin wing. The three-dimensional thick wing problem was studied by Djojodihardjo and Widnall,[12] who considered only dipoles distributed over the surface of a wing. Their Kutta condition consisted of two parts: 1) the strength of the trailing vortex was the difference between dipole strengths at the upper and lower surfaces of the trailing edge; and 2) the vortex-shedding velocity is determined by the average velocity vector at each pair of Kutta points where the dipole difference is calculated. Unfortunately, in the mathematical formulation by Djojodihardjo and Widnall,[12] the Fredholm integral equation is singular; a proper evaluation was made by using a limiting process.

The surface singularity method[1,2,7,12] has the capability of treating a wide variety of shapes. Unfortunately, in spite of its adaptability to complicated bodies, the surface-singularity method has difficulties in adjusting the body surface to a least-squares quadrilateral panel surface. Typically, the body is specified to a computer by the set of points which presumably lie exactly on the body surface; these eoints are arranged into groups of four "adjacent" points and a least-squares plane or a hyperboloidal element that passes through them. Two sources of error arise from the deviation between the approximate and the actual body. The errors are as follows:

1) The collocation points do not, in general, lie on the exact surface.

2) The computed surface normals are not exactly orthogonal to the true surface.

The latter one especially may cause severe error in certain cases, as mentioned by Hunt[2] and Webster.[13] Webster,[13] Johnson,[14] and Maskew[15] also examined these sources of errors for the steady-flow problem described by means of the surface-singularity method, either for the higher-order problem or for a geometry which required large computer storage.

In view of the complexity of the numerical management of the surface-singularity method, one asks the following question: Is there a method for calculating the flowfield past a three-dimensional, relatively uncomplicated body with less difficulty than the surface-singularity method? The method will be developed herein and will be called the interior-singularity panel method. The idea that will be developed is to select a plane running along the mean surface of the body and divide that plane into a number of two-dimensional panels. Source and dipole singularities will be placed on these panels and the strengths of the singularities will be determined. The nonpenetration boundary condition is satisfied on the control points which are located on the true body surface instead of the mean surface. Therefore the present method is also a extension of the linear theory to the moderately thick wing problem.

II. Kutta Condition

In Fig. 1, the plane labeled B represents the mean plane of the subject body, W represents the trailing vortex sheet, which

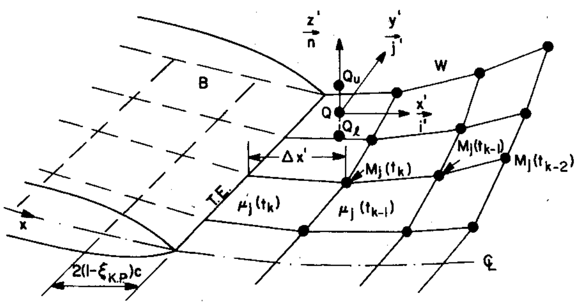

Fig. 1 Trailing edge and wake surface representation.

is of zero thickness and is supposed to sustain no pressure difference across it. The plane B is cut into panels and sources or sinks and dipole singularities are distributed over the panels. The trailing vortex sheet W is discretized by a set of connected quadrilateral dipole panels.

The velocity potential at any point P of the flowfield can be expressed by

$$\phi(P)=\frac{1}{4\pi}\left[\iint_B \sigma\left(\frac{1}{r}\right)\mathrm{d}S \iint_{B+W}\mu\left(\frac{z'(P)}{r^3}\right)\mathrm{d}S\right] \tag{1}$$

As shown in Fig. 1, $\boldsymbol{n}$ represents the local normal to the wake and intersects the wake at point Q. The points Q_u and Q_l lie on the $\boldsymbol{n}$ axis but on opposite sides of Q, and very close to each other.

Since there is no pressure difference across the wake plane, Bernoulli's equation can be written as

$$\tfrac{1}{2}[q^2(Q_l)-q^2(Q_u)]=\frac{\partial}{\partial t}[\phi(Q_u)-\phi(Q_l)] \tag{2}$$

Combining Eqs. (1) and (2) and letting Q_u approach Q_l, we have

$$\tfrac{1}{2}(q_l^2-q_u^2)=\frac{\partial}{\partial t}\left[\frac{1}{4\pi}\lim_{Q_u-Q_l-Q}\iint_{B+W}\mu\left(\frac{z'(Q_u)}{r^3}-\frac{z'(Q_l)}{r^3}\right)\mathrm{d}S\right]=\frac{\partial}{\partial t}\mu(Q) \tag{3}$$

where q_l, q_u are the abbreviation of $q(Q_l)$ and $q(Q_u)$.

Equation (3) represents a Kutta condition when the point Q is very close to the trailing edge and by discretizing the wake sheet and letting the strength of the dipoles be uniform within each wake panel. From Hess,[1] it is noted that the induced potential due to a dipole sheet of constant strength is equivalent to a vortex ring Γ around the periphery of the sheet with the same strength as the dipole, i.e., $\Gamma=\mu_Q$. Thus the time development of the circulation is expressed as

$$\tfrac{1}{2}(q_l^2-q_u^2)=\frac{\partial\Gamma}{\partial\tau} \tag{4}$$

From the Biot-Savart law, Eq. (4) can be reduced to the two-dimensional Kutta condition of Basu and Hancock,[8] which is restricted to the two-dimensional case. Basu and Hancock derived the Kutta condition from the circulation around a lifting body for a two-dimensional wing.

Without large computer storage, it will be difficult to get a converged result from Eq. (4). The reason is that convergence is not only based on a small increment of time but also on the location of the "Kutta point" where the velocity for Eq. (4) is calculated. The latter reason is the more difficult to achieve. This is due to the fact that as the Kutta point moves toward the trailing edge, the magnitude of the difference in $\boldsymbol{q}_l$ and $\boldsymbol{q}_u$ increases and requires a large computer storage to make the convergence test succeed. Therefore a different representation of the left-hand side of Eq. (4) is necessary.

From Milne-Thompson,[16] the surface vorticity and velocity difference on a vortex sheet are related by

$$\omega=-\boldsymbol{n}\times(\boldsymbol{q}_l-\boldsymbol{q}_u) \tag{5}$$

$$\boldsymbol{q}_l-\boldsymbol{q}_u=\boldsymbol{n}\times\omega \tag{6}$$

where $\boldsymbol{n}$ is the unit normal vector perpendicular to the local vortex sheet as shown in Fig. 1. Substituting Eq. (6) into Eq. (4) yields

$$\boldsymbol{q}_{\mathrm{av}}\cdot(\boldsymbol{n}\times\omega)=\frac{\partial\Gamma}{\partial\tau} \tag{7}$$

where q_{av} is the wake shedding velocity, defined as the average of q_u and q_l. Since the point Q is close to trailing edge, the average velocity off the body should be approximately the shedding velocity at Q. Thus the Kutta point can be chosen on the body but very close to the trailing edge. This procedure reduces the sensitivity to the wake inclination.

In the two-dimensional case, q_{av}, n, and ω are mutually perpendicular; therefore Eq. (7) can be reduced to the Kutta condition, as described by Giesing.[7]

From Eqs. (1) and (5) and the same limit logic as in Eq. (3), the surface vorticity on the infinitesimal vortex sheet is

$$\begin{aligned}\omega &= -n\, x\,(\mathrm{grad}\phi_l - \mathrm{grad}\phi_u) = -n\ x[\mathrm{grad}(\phi_l - \phi_u)] \\ &= n\, x\,(\mathrm{grad}\mu) = n\, x\left(\frac{\partial\mu}{\partial x'}i' + \frac{\partial\mu}{\partial y'}j'\right) \\ &= -\frac{\partial\mu}{\partial y'}i' + \frac{\partial\mu}{\partial x'}j' \end{aligned} \tag{8}$$

where x' and y' are the local coordinates lying on the wake element, and i' and j' are the unit vectors of the x', y' axes, as shown in Fig. 1. Substituting Eq. (8) into Eq. (7), and applying the result at the trailing edge (TE), the Kutta condition becomes

$$q_{av}\cdot\left(\frac{\partial\mu}{\partial x'}i + \frac{\partial\mu}{\partial y'}j\right) = -\frac{\partial\Gamma}{\partial t} \tag{9}$$

The Kutta condition is developed by avoiding the calculation of velocity difference at the trailing edge. Instead, the dipole singularity strength near the trailing edge is calculated and differentiated in the present study. In the numerical solution, the derivatives used in Eq. (9) are determined from finite differences.

III. Wake Surface

From Kelvin's theorem for an inviscid flow,

$$\frac{\mathrm{D}\Gamma}{\mathrm{D}\tau} = 0 \tag{10}$$

This means that once a circulation is generated for whatever reason, the strength remains constant as the fluid is convected downstream.

All the wake elements leaving the trailing edge are assumed to be quadrilateral, as shown in Fig. 1, where M_j represents the corner points of the wake elements, and the subscript j represents the corresponding strip. At the first dimensionless time $t_1\,(t = U_\infty \tau/c)$ downstream of the trailing edge, a set of wake elements is generated. Each of these elements is assumed to have a uniform strength of $\mu_j(t_1)$. From Hess,[1] the strength of circulation $\Gamma_j(t_1)\,[\Gamma_j(t_1) = \mu_j(t_1)]$ and location of the wake element $M_j(t_1)$ are found by an iteration procedure. Once the strength of circulation is determined from the complete field singularity strength and location by requiring convergence to within a specified tolerance, the perturbation velocity at location $M_j(t_1)$ is calculated, say $V[M_j(t_1)]$, and used to locate the corner point of each element from the position vector expression,

$$r[M_j(t_1)] = \tfrac{1}{2}t_1\{V[M_j(t_1)] + 0\} + t_1 U_\infty + r_{0,j} \tag{11}$$

The zero value is from the assumption that the perturbation velocity at the trailing edge is zero and $r_{0,j}$ is the position vector of the trailing edge for strip j. Some previous studies (see Ref. 17) suggest that the velocity needed to determine $M_j(t_k)$ in Fig. 1 should be on the location of the midpoint between $M_j(t_k)$ and the trailing edge. Labrujere[18] even suggests that the velocity at the center of the wake element could be computed and then extrapolated to the corner location. However, the accuracy attained for the size of the wake element should be suspect because of the extrapolation to obtain the velocity vector. After the first time step, the wake is considered to be convected downstream as a continuous surface sheet, and the circulation is kept constant. The kinematic condition of the wake surface at time t_k is

$$\begin{aligned} r[M_j(t_{k-n})] &= \tfrac{1}{2}\{V[M_j(t_{k-n})] \\ &\quad + V[M_j(t_{k-n+1})]\}[t_k - t_{k-1}] \\ &\quad + r[M_j(t_{k-n+1})] + U_\infty(t_k - t_{k-1}) \end{aligned} \tag{12}$$

for $n = 1,2,3,\ldots,k-1$.

At $n=0$, i.e., the row of wake elements nearest to the trailing edge, $V[M_j(t_{k+1})]$ in Eq. (12) is zero for the perturbation velocity at the trailing edge.

IV. Results and Discussion

The analysis of the preceding sections will now be applied to the problem of unsteady flow past the NACA 0015 wing with rectangular planform. The problem considered is the calculation of the flowfield past the wing due to a sudden change in incidence from 0 to 0.1 rad.

Numerically, the mean plane of the wing is divided into 15 strips with 14 elements for each strip, as is indicated in Fig. 1. Each of these panels is assumed to have a uniform singularity strength distributed over it. The size of the panels on the mean surface should be carefully examined. First, since distributed sources are used, the distance between the leading edge to the beginning of the first source location is a problem. Miloh and Landweber[19] discuss this problem for the two-dimensional case. In the present study on three-dimensional flows, the first source should begin at about 2% of the local chord length for a 24% thick wing. For wing thicknesses of 15% or less, it was found even that the first source could begin at the leading edge. The results are approximately the same for the thinner wings. Second, since each panel is distributed with uniform strength dipoles, the instability of the pressure coefficient on the control points on the body surface causes the lift solution to have a small degree of oscillation. Use of a higher-order dipole eliminates this slight instability. However, the use of a higher-order method could result in an unnecessary increase in computational cost for a slight increase in accuracy. Djojodihardjo and Widnall[12] commented on this point and observed that the uniform dipole description gave reasonably good results.

The nonpenetration boundary condition is

$$U_\infty \cdot n(P) = -n(P)\cdot \mathrm{grad}\phi \tag{13}$$

where $n(P)$ is the unit normal at the surface control point P. The control points are defined so that a line connecting the upper and lower points passes through the geometric center of the corresponding panel perpendicular to the panel. Thus, from Eqs. (1) and (13) and the procedure as described above, the governing equation becomes

$$\begin{aligned} -U_\infty\cdot n(P) &= \frac{1}{4\pi}\left[\Sigma\sigma\int_{\Delta S}\mathrm{grad}\left(\frac{1}{r}\right)\mathrm{d}S \right.\\ &\quad \left. + \Sigma\mu\int_{\Delta S}\mathrm{grad}\left(\frac{z}{r^3}\right)\mathrm{d}S\right]\cdot n(P) + \Sigma\Gamma V_\Gamma\cdot n(P) \end{aligned} \tag{14}$$

In this study, the singularity strength and location of the wake surface are determined as a part of the solution. Since V_Γ depends on the location of quadrilateral vortex ring [$r(M_j)$, as shown in Fig. 1], Eq. (14) is nonlinear.

Figure 2 shows a summary sketch of the wake geometry. Note the spanwise curvature of the trailing vortex sheet of the present study. The curvature is similar to what measurements

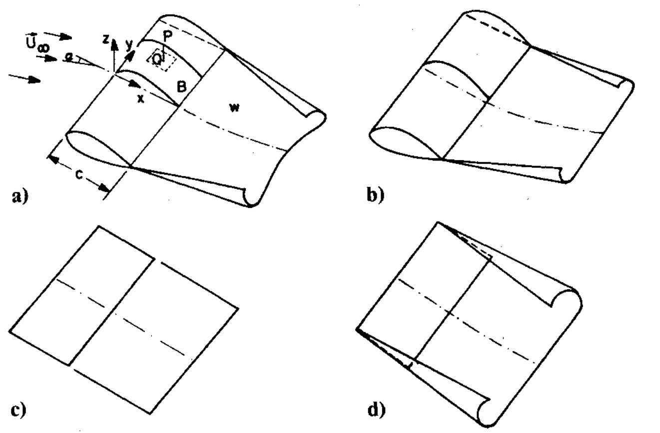

Fig. 2 Comparison of the rolling up of the wake sheet from different methods: a) present study; b) Ref. 12; c) Ref. 9; and d) Ref. 10.

show and to what Jepps[17] indicated for an infinitesimally thin wing.

In the chordwise direction, the wake velocity field has the following general behavior: For small times, the wake surface has an upward motion tendency, but as time increases the wake vortices cause mutually induced velocities which drive the wake near the trailing edge down and allow the downstream edge of the wake to curl up.

The influence of the x-axis location of the Kutta point ξ_{KP} and the time increment Δt to the circulation $\Delta\Gamma$ is stated as follows:

1) For fixed Δt, the Kutta point moves toward the TE, i.e., $(1-\xi_{KP})$ decreases, the values of $\Delta\Gamma$ from Eq. (9) are decreased.

2) For a fixed location of the Kutta point, decreasing Δt causes $\Delta\Gamma$ to increase.

Thus, in order to get a converged result [which means both Δt and $(1-\xi_{KP})$ are small enough to get an accurate $\Delta\Gamma$], the time increment is chosen to be related to the distance between the TE and the KP,

$$\Delta t = (t_k - t_{k-1}) = C_l(1-\xi_{KP}) \tag{15}$$

The determination of C_l in Eq. (15) is as follows. In Eq. (9) both time and distance derivatives are represented by finite quantities. Two questions are posed. As shown in Fig. 1, should we choose $\delta x = \frac{1}{2}[2(1-\xi_{KP})C + \Delta x']$ for the reason that dipole strength is represented at the centroid of each quadrilateral panel (before the TE and also after the TE). Or, should we choose $\delta x = Rx'$, as shown in Fig. 1, for the reason that Eq. (9) is derived for the wake. If we give equal weight to each explanation, a solution is to accept both and equate the two values of δx, which gives

$$\delta x = \tfrac{1}{2}[2(1-\xi_{KP})C_l + \Delta x'] = \Delta x' \tag{16}$$

This simplifies to

$$\Delta x' = 2(1-\xi_{KP})C_l \tag{17}$$

Since the wake shedding velocity is approximately U_∞, a nondimensional time increment to shed a wake of a length $\Delta x'$ is about

$$\Delta t = \frac{\Delta x'}{U_\infty}\,\frac{U_\infty}{C} = 2(1-\xi_{KP}) \tag{18}$$

This is also consistent with the statement of Djojodihardjo and Widnall[12] that "the time step is commensurate with the spacing of the element along the chord."

The chordwise dipole strength variation at $\eta = 0$ and 0.9375 is shown in Fig. 3 for the steady flow case and for unsteady flow at a dimensionless time of 0.12. The calculated singularity strengths at the center of each panel are indicated for each curve. The dipole strength behind the trailing edge ($x/c > 1$) is actually the strength of the trailing vortex because each wake element has constant dipole strength, as stated earlier.

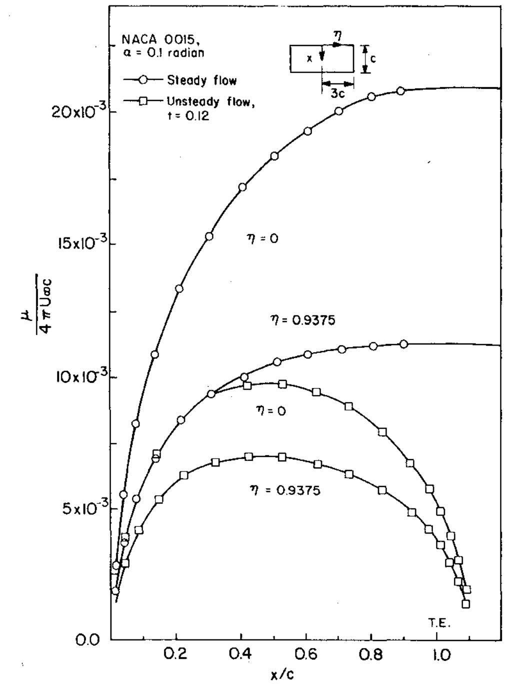

Fig. 3 Chordwise dipole strength variation.

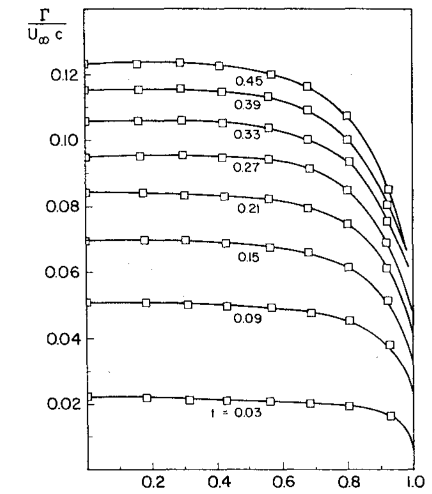

Fig. 4 Spanwise circulation variation.

The slope of μ at the trailing edge is used to determine the strength of the shed vorticity from Eq. (8) by means of finite differences. Thus we have

$$\omega = -\frac{\Delta\mu}{\Delta y'}\,i + \frac{\Delta\mu}{\Delta x'}\,j \tag{19}$$

evaluated at the trailing edge.

Since we have represented the trailing-edge vorticity by means of finite differences, we need to examine qualitatively the accuracy of the approximation. It is important to have as accurate a value of ω as is practical since the trailing edge vorticity leads to the time derivative of the circulation. First, we consider the chordwise variation of the dipole strengths.

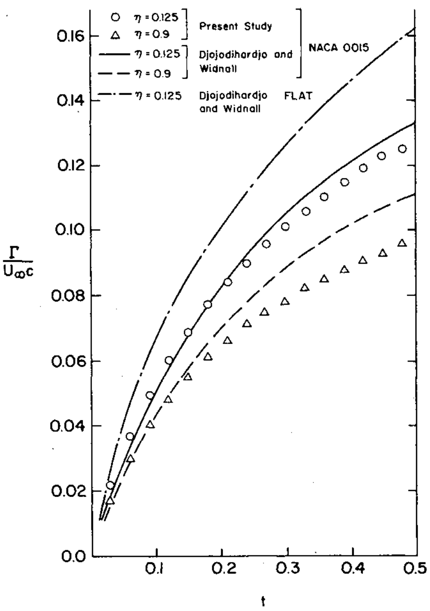

Fig. 5 Time development of circulation.

Fortunately, as seen in Fig. 3, the slope of the chordwise dipole-strength variations is reasonably constant at the trailing edge for both steady and unsteady flow. At a dimensionless time of 0.12, the line connecting the two points just on either side of the trailing edge has very little curvature. The lack of curvature indicates that the finite-difference representation of the slope should very closely approximate the actual slope. Therefore we are satisfied that the finite-difference representation in the chordwise direction is sufficiently accurate.

For the steady flow case, the dipole strength at the trailing edge becomes constant in the chordwise direction. Therefore the finite-difference representation of the slope produces a zero value which is what is expected for the exact case. Thus we have reasonable estimates of the chordwise contribution to the trailing edge vorticity.

Figure 4 shows the development of circulation in the spanwise direction for various nondimensional times. This is important in the determination of the trailing-edge vorticity. Again, we perform a qualitative evaluation of the slope of the dipole strength, this time in the spanwise direction. As is obvious in Fig. 4, the spanwise variation for small time is practically zero for small values of η. At a given time and large η, the spanwise contribution is no longer practically zero, but is still much smaller than the chordwise variation for our case of an aspect ratio of 6.

As time increases, the spanwise variation of the dipole becomes larger and the chordwise variation becomes smaller at the trailing edge. As steady flow is approached, the chordwise variation of the dipole strength vanishes as discussed earlier. Based on these results for the developing flow, the major contribution of trailing edge vorticity to the circulation is in the spanwise direction. For steady flow, the major component of vorticity is in the chordwise direction. We also suggest that the Kutta condition given by Eq. (9) is better for small time while Eq. (4) is a better approximation for large times.

Our time development of circulation is compared to that of Djojodihardjo and Widnall[12] in Fig. 5. The development is shown at two different spanwise locations. At $\eta = 0.125$, the present results show that the circulation develops only slightly less rapidly than the results of Ref. 12 indicate. For $\eta = 0.9$, there is a greater difference in the development of circulation with the present results still being less. It would be difficult to determine which of these results is more representative of the actual flow. Near the wing-tip region, where a large change of circulation occurs, it must be noted that the insufficiency of number of panels and the neglect of viscous effects on the calculation of the rolling up of the wing-tip vortex mean that the present results and those of Ref. 12 should be interpreted with some reservation. The present result is also compared with the result of the zero thickness wing at $\eta = 0.125$. Figure 5 shows that the circulation grows more slowly as the airfoil becomes thicker. This also demonstrates the importance of the present method as a modified classic linear theory for the unsteady flow case.

V. Conclusions

We have demonstrated that the interior-singularity panel method has the capability of representing the time-dependent flow past a moderately thick wing. Because of the ease of manipulating the panels, we feel that the interior-panel method is less complicated than the surface methods used by other investigators. An alternate method to that of Basu and Hancock,[8] requiring less computer storage, was developed to describe the Kutta condition.

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